

# PREFACE

*Une totale soumission aux données de l'expérience  
est la règle d'or qui domine toute discipline,  
toute activité valable.*

Maurice Allais  
lycée Lakanal  
February 3, 2001

These notes are about empirical finance, and focus on the pricing and risk management of financial assets: bonds, futures contracts, and other derivative securities.

Following Maurice Allais's advice, the emphasis is empirical. We present models, discuss their implementation, and verify their relevance by testing them on real data. Throughout the text, we emphasize:

- an incremental approach to model building, starting from simple models, and building upon that foundation to construct more complex models, as needed;
- a data-driven approach: we will implement all the models that are presented, using the R statistical package and the Rmetrics libraries;
- the systematic use of simulation as a way of validating modeling decisions or measuring various modeling biases.

This text would not be possible without the R statistical program and without the numerous Rmetrics packages. We extend our deep appreciation to the R community and to Diethelm Würtz and the Rmetrics team.

Finally, these notes grew out of courses in computational finances given at Télécom-Bretagne and EURIA (Université de Bretagne Occidentale), both situated in Brest, France, as well as at IAE-Paris (Université Paris I Panthéon-Sorbonne). The students there deserve a special thank you for enduring early drafts of the manuscript.

Patrick Hénaff  
St Jean Trolimon, Winter 2012

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